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Approximate Weinberg Mixing

Philip D. Mannheim[1]

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We present a model of the weak interactions in which a custodial symmetry that is not an invariance of the starting Lagrangian emerges in the effective low-energy sector of the theory. This symmetry maintains the relation $M_W = M_Z \cos\theta_W$ to all orders in the Higgs self-couplings to any required degree of accuracy, while leaving the quark mass spectrum completely unconstrained. The model is a local left-right symmetric chiral flavor gauge theory of the electroweak interactions in which the symmetry is spontaneously broken by fundamental Higgs fields which transform the same way under the chiral group as fermion Dirac and Majorana masses.

1. INTRODUCTION

The observation of neutral currents established the validity of the Weinberg-Salam-Glashow theory of weak interactions (Weinberg, 1967; Salam, 1968; Glashow, 1961). The experimental data are conveniently characterized by a parameter

$$\rho = \frac{M_W}{M_Z \cos\theta_W} \tag{1}$$

which measures the relative strengths of the charged and neutral currents. Here θ_W is the Weinberg angle which determines the amount of mixing between the two neutral gauge bosons of the local $SU(2)_L \times U(1)_{WS}$ gauge theory. Experimentally ρ is found to be very close to 1. The very fact that ρ is of order 1 rather than say of order $1/\alpha$ already indicates that the neutral currents are indeed on an equal footing with the charged ones, with the explicit value of ρ then giving detailed information about the pattern of spontaneous symmetry breaking in the model.

In the standard model approach, for instance, where the $SU(2)_L \times U(1)_{WS}$ symmetry is broken by a single complex $SU(2)_L$ Higgs doublet,

[1]Department of Physics, University of Connecticut, Storrs, Connecticut 06268.

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ϕ_a, the pure Higgs sector contribution yields for ρ the value 1, thus making the standard model breaking pattern both simple and in accord with experiment. The reason that this particular value for ρ is obtained in the standard model was identified by Susskind (1979) and by Weinberg (1976, 1979). They noted that the most general Higgs potential, $V(\phi_a)$, for a single complex Higgs doublet just happens to possess the higher six-parameter $O(4)$ symmetry of a real quartet of fields. With one of these real fields acquiring a vacuum expectation value the Higgs potential can only produce three Goldstone bosons. Thus after the breaking the potential must still possess a residual unbroken three-parameter global $SU(2)$ symmetry. Under this residual or custodial $SU(2)$ symmetry the three $SU(2)_L$ intermediate vector bosons transform as a triplet, so that the Higgs self-coupled sector (both tree approximation and radiative corrections) gives these three vector bosons degenerate Higgs mechanism masses. The mixing of the electrically neutral $SU(2)_L$ vector boson with the $U(1)_{WS}$ gauge boson then yields $\rho = 1$ for the all order pure Higgs sector contribution.

The higher $O(4)$ invariance of the input Higgs potential and the residual global $SU(2)$ symmetry that survives the Higgs breaking are only exact as far as the Higgs sector is concerned, with both symmetries being broken in the couplings of the Higgs fields to the gauge bosons and fermions of the model. Hence the gauge boson and fermion contributions cause ρ to deviate from 1. Fortunately, the gauge boson loop corrections start off in order α and are hence small, while, with the Yukawa couplings being of order $m_f/\langle\phi_a\rangle$, i.e., of order $em_f/M_W \sin\theta_W$, the currently known fermions only in fact give small contributions to ρ. Thus in the standard model the value of ρ is protected not just by the custodial $SU(2)$ symmetry in the Higgs sector but also by the phenomenologically small fermionic contribution.

To see just how sensitive the value of ρ is to the details of the symmetry-breaking mechansim we consider instead an $SU(2)_L \times U(1)_{WS}$ model which possesses not one but two fundamental complex Higgs doublets, ϕ_a and ϕ_b, say. Now the $O(4)$ invariance of the ϕ_a self-coupled sector is broken directly in its couplings to ϕ_b so that the most general Higgs potential is only $SU(2)_L \times U(1)_{WS}$ invariant and possesses no further symmetry at all. Despite the absence of any such additional symmetry it turns out that the tree approximation Higgs sector contribution still yields $\rho = 1$. In higher orders, however, where the induced trilinear Higgs couplings are of order $eM_H^2/M_W \sin\theta_W$ (M_H being a typical Higgs mass), and where the lowest-order Higgs field wave function renormalization constant behaves like M_H^{-2}, the radiative corrections to ρ start out in order $e^2M_H^2/M_W^2 \sin^2\theta_W$, to give a value for ρ for which there is no known phenomenological bound. Moreover, we see that since the trilinear vertices grow with M_H^2, heavy Higgs bosons do not only not decouple from ρ, but rather they actually cause ρ to grow with M_H^2 (i.e., with the Higgs quartic self-coupling constants) in a potentially uncontrollable manner.

A slightly more constrained and commonly considered model with two complex $SU(2)_L$ Higgs doublets is that suggested by dynamical symmetry breaking, namely, the chiral flavor model in which the Higgs sector possesses a full global $SU(2)_L \times SU(2)_R \times (B-L)$ invariance (B and L are, respectively, the baryon and lepton number generators). This model possesses an $SU(2)_L \times U(1)_{WS}$ subgroup where

$$U(1)_{WS} = T_3^R + (B-L)/2 \tag{2}$$

and also an electromagnetic subgroup with generator

$$J_{\text{em}} = T_3^L + T_3^R + (B-L)/2 \tag{3}$$

The symmetry breaking is effected by a fundamental Higgs field χ which transforms according to the $(\mathbf{2}, \mathbf{2}^*, \mathbf{0}) \oplus (\mathbf{2}^*, \mathbf{2}, \mathbf{0})$ representation of the chiral flavor group, i.e., like a fermion Dirac mass. The Higgs field χ contains 8 real components which can be written as 2 real irreducible $O(4)$ quartets, viz.,

$$\begin{aligned} \alpha &= (\sigma_0, \pi_1, \pi_2, \pi_3) \\ \beta &= (\pi_0, \sigma_1, \sigma_2, \sigma_3) \end{aligned} \tag{4}$$

(in the notation of the convenient σ model), or as two complex $SU(2)_L$ doublets, viz.,

$$\phi_a = \begin{pmatrix} \sigma_0 + i\pi_3 \\ i\pi_1 - \pi_2 \end{pmatrix}, \qquad \phi_b = \begin{pmatrix} \sigma_3 + i\pi_0 \\ \sigma_1 + i\sigma_2 \end{pmatrix} \tag{5}$$

In the event that the Higgs sector breaking is such that σ_0 is the only component of χ that acquires a vacuum expectation value, isospin remains unbroken and a residual global $SU(2)_{L+R} \times (B-L)$ invariance survives the breaking. Giving a local extension to the $SU(2)_L \times U(1)_{WS}$ subgroup of $SU(2)_L \times SU(2)_R \times (B-L)$ then provides just the right number of gauge bosons with just the right quantum numbers to absorb the three Goldstone bosons produced by the breaking pattern, with the residual unbroken global $SU(2)_{L+R}$ invariance then enforcing $\rho = 1$ to all orders in the Higgs potential.

Characteristic of this mechanism is the presence of an additional global $SU(2)$ invariance [either isospin or some other appropriate $SU(2)$ (Sikivie et al., 1980)] already at the level of the input Higgs Lagrangian. Moreover, as we can see, the mechanism requires a rather delicate interplay between the local and global sectors of the theory. First, the symmetry-breaking potential must possess a global symmetry larger than that of the local gauge sector of the theory, and second the symmetry of the potential must only be partially broken spontaneously so as to produce just the right number of Goldstone bosons with just the right set of quantum numbers required to give Higgs mechanism masses to just the right set of gauge bosons; while finally just the right residual global symmetry needed to enforce $\rho = 1$ must be left unbroken.

Apart from requiring this detailed interplay between the local and global sectors of the theory, the residual symmetry of this mechanism can also give rise to potentially undesirable mass formulas. In the simplest case, for instance, where the residual $SU(2)$ is in fact isospin, the residual symmetry enforces the phenomenologically undesirable degeneracy of the up and down quarks. Moreover, there are many current models of the electroweak interactions which do not possess any such extra symmetry in the starting Lagrangian; and also others, such as the popular $SO(10)$ grand-unified model of the strong, electromagnetic, and weak interactions, which while possessing a full $SU(2)_L \times SU(2)_R \times (B-L)$ invariance, do so at the local rather than at the global level, so that the $SU(2)_{L+R}$ group is then necessarily spontaneously broken.

While much attention in the literature has been given to the problem of the quark mass degeneracy in models whose input Lagrangians possess an additional global symmetry which then survives the breaking (Sikivie et al., 1980; Dimopoulos and Susskind, 1979; Eichten and Lane, 1980; Dimopoulos et al., 1980), the question of the value of ρ in models without such additional input symmetries in the first place (a common phenomenological situation) has not been adequately addressed. In this paper we shall resolve this latter question by presenting a model in which the full symmetry of the Higgs potential is the same as that of the local gauge sector of the theory, and in which the gauge sector (and hence the Higgs sector) is fully broken all the way down to electromagnetism. In the model an effective residual symmetry which is not an invariance of the starting Lagrangian will emerge in the low-energy structure of the theory which will then keep ρ close to 1 while imposing no mass constraints on the fermions. Additional input global symmetries are thus not needed in order to obtain $\rho = 1$.

The essence of our approach is to note that a single complex Higgs doublet, ϕ_a, say, always possesses a higher $O(4)$ symmetry in its own self-couplings (this we get for free as it were), with this higher symmetry then being broken at the level of the input Lagrangian in the couplings of ϕ_a to the other Higgs fields of the model. If these other Higgs fields could be given large masses by the symmetry-breaking mechanism in a way which would simultaneously decouple all these other Higgs fields not just from ϕ_a but also from the Weinberg–Salam gauge boson mass matrix, the higher ϕ_a self-coupling $O(4)$ symmetry would then be able to emerge and control the effective low-energy structure of the theory. In this way the theory thus generates its own custodial symmetry and none need be supplied externally by hand by giving the Higgs sector some additional input symmetry.

The requisite model which we shall present in this paper is the popular left-right symmetric local chiral flavor gauge theory of the electroweak interactions in which both the Higgs and gauge sectors are $SU(2)_L \times SU(2)_R \times (B-L)$ invariant. The symmetry will be broken by fundamental Higgs fields which transform the same way under the chiral group as fermion bilinear Dirac and Majorana masses. We will find that the radiative corrections in this model are structured in a way which permits ρ to only deviate from 1 by a small controllable amount dependent on the relative strengths of the left-handed and right-handed currents of the local chiral theory (a strength ratio which is known to be small phenomenologically), with fermion mass ratios being completely unconstrained.

Some of our results have already been presented in a note (Mannheim, 1983), and in this paper we give the details. The present paper is organized as follows. We study first, in Sections 2 and 3, theories in which the local gauge sector is $SU(2)_L \times U(1)_{WS}$ invariant, but in which the associated Higgs sector is built out of fundamental Higgs fields which transform as fermion Dirac masses and possesses the larger global $SU(2)_L \times SU(2)_R \times (B-L)$ invariance. Unlike the situation considered in Susskind (1979) and Weinberg (1976, 1979) we shall break the Higgs sector symmetry down beyond isospin so that no residual $SU(2)$ survives the breaking. Then the fermions are nondegenerate while ρ of course deviates arbitrarily from 1. Nonetheless we find (Section 2) that ρ does in fact equal 1 in the tree approximation despite the absence of any residual symmetry, so that it only deviates from 1 in higher orders (Section 3). Apart from thus failing to given an acceptable value for ρ, the model is also unsatisfactory in that the potential generates more Goldstone bosons than can be absorbed by the Weinberg-Salam gauge bosons. To eliminate these additional Goldstone bosons from the physical spectrum we must thus enlarge the gauge sector so that it then also possesses the chiral $SU(2)_L \times SU(2)_R \times (B-L)$ invariance of the potential. To break this larger, parity-conserving theory we augment the Higgs sector by introducing parity violating fundamental Higgs fields which transform like fermion Majorana masses. In Section 4 we show how to break the local chiral theory so as to yield ρ of order 1 in the tree approximation, again without the need for a residual symmetry. Then finally, in Section 5, we study the radiative corrections in the local chiral model to find that this time there is an effective residual symmetry generated in the low-energy sector of theory which keeps ρ close to 1 as required with the fermion masses being completely unconstrained.

2. THE WEINBERG–SALAM MODEL WITH A CHIRAL HIGGS POTENTIAL IN THE TREE APPROXIMATION

We begin our analysis with some general remarks on symmetry breaking. We consider the gauge boson sector of the standard $SU(2)_L \times U(1)_{WS}$ Weinberg–Salam theory, viz. (dropping Lorentz indices),

$$\mathscr{L} = gW_1^L(V_1 - A_1) + gW_2^L(V_2 - A_2) + gW_3^L(V_3 - A_3) + g' W_{WS}(J_{\rm em} - V_3 + A_3) \tag{6}$$

Here V_i and A_i $(i = 1, 2, 3)$ are, resepectively, vector and axial-vector currents, W_i^L $(i = 1, 2, 3)$ form an $SU(2)_L$ triplet of vector bosons, W_{WS} couples to the $U(1)_{WS}$ current of equation (2), and $J_{\rm em}$ is the electromagnetic current of equation (3). We introduce new basis states for the neutral sector

$$A = -\frac{(g' W_3^L + gW_{WS})}{(g^2 + g'^2)^{1/2}}$$
$$Z_L = \frac{gW_3^L - g' W_{WS}}{(g^2 + g'^2)^{1/2}} \tag{7}$$

In terms of this basis we can rewrite the coupling of the gauge bosons to the currents as

$$\mathscr{L} = gW_1^L(V_1 - A_1) + gW_2^L(V_2 - A_2) - g \sin \theta_W(A + \tan \theta_W Z_L)J_{\rm em} + g \sec \theta_W Z_L(V_3 - A_3) \tag{8}$$

where we have introduced the Weinberg angle via

$$\tan \theta_W = \frac{g'}{g} \tag{9}$$

After the symmetry is broken spontaneously the potential will produce three Goldstone bosons L_i $(i = 1, 2, 3)$ of interest which couple to the left-handed currents with strengths

$$\langle 0|(V_i - A_i)_\lambda|L_i\rangle = iq_\lambda F_i^L \tag{10}$$

Consequently the Higgs mechanism yields a set of gauge bosons with mass matrix

$$M = \tfrac{1}{2}g^2(W_1^L)^2(F_1^L)^2 + \tfrac{1}{2}g^2(W_2^L)^2(F_2^L)^2 + \tfrac{1}{2}g^2 \sec^2 \theta_W Z_L^2(F_3^L)^2 \tag{11}$$

to lowest order in the gauge-coupling constants but to all orders in the symmetry-breaking potential.

The above analysis is completely general and only depends on the fact that each broken $SU(2)_L \times U(1)_{WS}$ current has an associated Goldstone boson. The analysis is independent of whether the symmetry is broken dynamically or by fundamental Higgs fields, is independent of the initial symmetry of the potential, and is independent of how the various Goldstone bosons transform according to the symmetry group of the potential. Further restrictions come when we specify the group structure of the breaking pattern. We are interested here in the case where the symmetry-breaking potential is $SU(2)_L \times SU(2)_R \times (B-L)$ invariant with the breaking being according to the $(\mathbf{2}, \mathbf{2}^*, \mathbf{0}) \oplus (\mathbf{2}^*, \mathbf{2}, \mathbf{0})$ representation, χ, of the chiral group. It is convenient to reexpress χ in a σ model basis as

$$\chi = \frac{1}{\sqrt{2}}[\sigma_0 + i\pi_0 + (\bar{\sigma} + i\bar{\pi}) \cdot \bar{\tau}] \tag{12}$$

which contains two real irreducible quartets, $\alpha = (\sigma_0, \pi_1, \pi_2, \pi_3)$ and $\beta = (\pi_0, \sigma_1, \sigma_2, \sigma_3)$. Classifying according to the $SU(2)_L \times U(1)_{WS}$ subgroup we see that χ consists of two complex $SU(2)_L$ doublets each with a $U(1)_{WS}$ quantum number of 1/2. From the standard current algebra commutators we obtain the following useful relations:

$$\begin{aligned} [V_1 - A_1, \langle\sigma_0\rangle\pi_1 + \langle\sigma_3\rangle\sigma_2] &= [V_2 - A_2, \langle\sigma_0\rangle\pi_2 - \langle\sigma_3\rangle\sigma_1] \\ &= [V_3 - A_3, \langle\sigma_0\rangle\pi_3 + \langle\sigma_3\rangle\pi_0] \\ &= i\langle\sigma_0\rangle\sigma_0 + i\langle\sigma_3\rangle\sigma_3 \end{aligned} \tag{13}$$

We define the couplings of the Goldstone bosons L_i to these elements of χ to be

$$\begin{aligned} \langle L_1|\langle\sigma_0\rangle\pi_1 + \langle\sigma_3\rangle\sigma_2|0\rangle &= Z_1^{1/2}[\langle\sigma_0\rangle^2 + \langle\sigma_3\rangle^2]^{1/2} \\ \langle L_2|\langle\sigma_0\rangle\pi_2 - \langle\sigma_3\rangle\sigma_1|0\rangle &= Z_2^{1/2}[\langle\sigma_0\rangle^2 + \langle\sigma_3\rangle^2]^{1/2} \\ \langle L_3|\langle\sigma_0\rangle\pi_3 + \langle\sigma_3\rangle\pi_0|0\rangle &= Z_3^{1/2}[\langle\sigma_0\rangle^2 + \langle\sigma_3\rangle^2]^{1/2} \end{aligned} \tag{14}$$

Saturating the vacuum expectation values of the relations of equation (13) with these Goldstone bosons yields

$$F_1^L Z_1^{1/2} = F_2^L Z_2^{1/2} = F_3^L Z_3^{1/2} = [\langle\sigma_0\rangle^2 + \langle\sigma_3\rangle^2]^{1/2} \tag{15}$$

Thus the mixing parameter ρ is given by

$$\rho = \frac{M(W_1^L)}{\cos\theta_W M(Z_L)} = \frac{F_1^L}{F_3^L} = \left(\frac{Z_3}{Z_1}\right)^{1/2} \tag{16}$$

and is now expressed in terms of the quantities Z_i which refer explicitly to the $(\mathbf{2}, \mathbf{2}^*, \mathbf{0}) \oplus (\mathbf{2}^*, \mathbf{2}, \mathbf{0})$ representation. Again the relation is exact to all orders in the potential and independent of whether the symmetry breaking is dynamical or fundamental.

In terms of the above language the result of Susskind (1979) and Weinberg (1976, 1979) is simply the statement that when isospin is unbroken (i.e. when $\langle\sigma_3\rangle = 0$) Z_1 and Z_3 are equal, so $\rho = 1$. With the isospin being unbroken the fermions have to acquire degenerate masses. The starting point of our present study is to note that the above analysis does not tell us by how much the gauge boson and fermion mass formulas deviate from their symmetry values when the isospin symmetry is not exact. According to the Goldberger–Treiman relations the fermion masses are related to the various F_i^L by factors depending on the coupling constants of the L_i Goldstone bosons to the fermions. When isospin is broken these coupling constants are not equal. Thus the deviation of ρ from 1 is given by the deviation of Z_1 from Z_3, while the deviation of the fermion masses from exact degeneracy depends on the deviation of the various Goldstone boson to fermion coupling constants from each other. Thus we can decouple the value of ρ from the fermion mass ratio since the deviations of the Z_i and the deviations of the Goldstone boson to fermion coupling constants can in principle be uncorrelated.

The simplest situation in which this explicitly happens is the tree approximation to a σ model with fundamental Higgs fields. Specifically, in the σ model itself the quantities Z_1 and Z_3 are wave-function renormalization constants, and the fields in α and β are the Goldstone bosons themselves. In the tree approximation there is no renormalization so that Z_1 and Z_3 both equal 1. Consequently Z_1 and Z_3 are equal to each other and thus $\rho = 1$. Moreover it is straightforward to construct a tree approximation minimum to the most general fundamental $SU(2)_L \times SU(2)_R \times (B-L)$ invariant Higgs potential

$$V(\chi) = -a \operatorname{Tr} \chi^+\chi + b(\operatorname{Tr} \chi^+\chi)^2 + c \operatorname{Tr}(\chi^+\chi)^2 + r[|\det \chi|^2 + |\det \chi^+|^2] \tag{17}$$

in which both $\langle\sigma_0\rangle$ and $\langle\sigma_3\rangle$ are nonzero. Since the analysis leading to equation (16) did not depend on the specific values of $\langle\sigma_0\rangle$ and $\langle\sigma_3\rangle$ we see that in the tree approximation $\rho = 1$ despite the absence of any residual $SU(2)_{L+R}$ symmetry. The fermion mass ratio is thus completely unconstrained in the tree approximation, and hence decoupled from the value of ρ.

It is instructive to derive the above result in a slightly different manner. In the σ model we can express the currents directly in terms of the fields in α and β, viz.,

$$\begin{aligned}(V_1-A_1)_\lambda &= -\sigma_0\overleftrightarrow{\partial}_\lambda\pi_1+\pi_2\overleftrightarrow{\partial}_\lambda\pi_3+\pi_0\overleftrightarrow{\partial}_\lambda\sigma_1+\sigma_2\overleftrightarrow{\partial}_\lambda\sigma_3\\ (V_2-A_2)_\lambda &= -\sigma_0\overleftrightarrow{\partial}_\lambda\pi_2+\pi_3\overleftrightarrow{\partial}_\lambda\pi_1+\pi_0\overleftrightarrow{\partial}_\lambda\sigma_2+\sigma_3\overleftrightarrow{\partial}_\lambda\sigma_1\\ (V_3-A_3)_\lambda &= -\sigma_0\overleftrightarrow{\partial}_\lambda\pi_3+\pi_1\overleftrightarrow{\partial}_\lambda\pi_2+\pi_0\overleftrightarrow{\partial}_\lambda\sigma_3+\sigma_1\overleftrightarrow{\partial}_\lambda\sigma_2\end{aligned} \tag{18}$$

Defining

$$L_1=\frac{\langle\sigma_0\rangle\pi_1+\langle\sigma_3\rangle\sigma_2}{[\langle\sigma_0\rangle^2+\langle\sigma_3\rangle^2]^{1/2}} \tag{19}$$

and analogously for L_2 and L_3 then yields

$$(V_1-A_1)_\lambda\sim-[\langle\sigma_0\rangle^2+\langle\sigma_3\rangle^2]^{1/2}\partial_\lambda L_1 \tag{20}$$

etc. after translating to the tree approximation minimum, so that

$$F_1^L=F_2^L=F_3^L=[\langle\sigma_0\rangle^2+\langle\sigma_3\rangle^2]^{1/2} \tag{21}$$

for any values of $\langle\sigma_0\rangle$ and $\langle\sigma_3\rangle$. As can be seen from equation (18) the α and β contributions are decoupled from each other in the V_i-A_i currents and thus act as two separate complex $SU(2)_L$ doublets. Equation (21) is then just the well-known result that two complex doublets (or any number for that matter) always give ideal $\rho=1$ mixing in the tree approximation to the Weinberg-Salam theory, independent of their relative orientation to each other.

Though we have now obtained a nice limit in which $\rho=1$ without a residual symmetry, in order to take advantage of it we must investigate the stability of the result under radiative corrections. As we shall now show the above σ model is unfortunately unable to maintain the relation $\rho=1$ in higher orders. (When $\langle\sigma_0\rangle\neq\langle\sigma_3\rangle$ the above model also produces two more Goldstone bosons than can be absorbed by the Weinberg-Salam gauge bosons, a point we shall resolve below in Section 4.)

3. RADIATIVE CORRECTIONS TO THE WEINBERG-SALAM MODEL WITH A CHIRAL HIGGS POTENTIAL

In this section we investigate the stability of the tree approximation analysis against radiative corrections. Rather than perturb around the most general σ_0, σ_3 breaking pattern we can obtain the result we require by working in a simplified situation in which $\langle\sigma_3\rangle=-\langle\sigma_0\rangle$ in tree approximation (the fermions are then nondegenerate, though one is kept massless). This will not change the nature of our results but will make the calculations more straightforward. In terms of the fields in α and β we define a new basis

$$\begin{aligned}
\sigma_+ &= \frac{1}{\sqrt{2}}(\sigma_0+\sigma_3), & \pi_+ &= \frac{1}{\sqrt{2}}(\pi_0+\pi_3) \\
\sigma_- &= \frac{1}{\sqrt{2}}(\sigma_0-\sigma_3), & \pi_- &= \frac{1}{\sqrt{2}}(\pi_0-\pi_3) \\
\sigma_A &= \frac{1}{\sqrt{2}}(\sigma_1+\pi_2), & \pi_A &= \frac{1}{\sqrt{2}}(\pi_1-\sigma_2) \\
\sigma_B &= \frac{1}{\sqrt{2}}(\sigma_1-\pi_2), & \pi_B &= \frac{1}{\sqrt{2}}(\pi_1+\sigma_2)
\end{aligned} \tag{22}$$

so that we can reexpress the potential $V(\chi)$ of equation (17) as

$$\begin{aligned}
V(\chi) = &-a[\sigma_+^2+\sigma_-^2+\pi_+^2+\pi_-^2+\sigma_A^2+\sigma_B^2+\pi_A^2+\pi_B^2] \\
&+(b+c)[\sigma_+^2+\sigma_-^2+\pi_+^2+\pi_-^2+\sigma_A^2+\sigma_B^2+\pi_A^2+\pi_B^2]^2 \\
&-2(c-r)[\sigma_+\sigma_- - \pi_+\pi_- - \sigma_A\sigma_B+\pi_A\pi_B]^2 \\
&-2(c+r)[\sigma_+\pi_- + \sigma_-\pi_+ - \sigma_A\pi_B - \sigma_B\pi_A]^2
\end{aligned} \tag{23}$$

We translate σ_- by p where $2(b+c)p^2=a$ and find a tree approximation minimum in which $\langle\sigma_-\rangle=p$, $\langle\sigma_+\rangle=0$. After translating, the complete quadratic term of $V(\chi)$ is given by

$$V_{\text{Quad}}(\chi)=4(b+c)p^2\sigma_-^2-2(c-r)p^2\sigma_+^2-2(c+r)p^2\pi_+^2 \tag{24}$$

Thus σ_-, σ_+, and π_+ will acquire positive squared masses if $b+c$, $-c+r$, and $-c-r$ are all positive. Hence our minimum is natural in the sense that it can be obtained for a continuous range of parameters. In the minimum π_A, σ_A, π_-, π_B, and σ_B are all massless. We rewrite the currents of equations (18) in the new basis

$$\begin{aligned}
(V_1-A_1)_\lambda &= -\sigma_+\overleftrightarrow{\partial}_\lambda\pi_B+\pi_+\overleftrightarrow{\partial}_\lambda\sigma_B-\sigma_-\overleftrightarrow{\partial}_\lambda\pi_A+\pi_-\overleftrightarrow{\partial}_\lambda\sigma_A \\
(V_2-A_2)_\lambda &= \sigma_+\overleftrightarrow{\partial}_\lambda\sigma_B+\pi_+\overleftrightarrow{\partial}_\lambda\pi_B-\sigma_-\overleftrightarrow{\partial}_\lambda\sigma_A-\pi_-\overleftrightarrow{\partial}_\lambda\pi_A \\
(V_3-A_3)_\lambda &= -\sigma_+\overleftrightarrow{\partial}_\lambda\pi_++\sigma_-\overleftrightarrow{\partial}_\lambda\pi_--\sigma_A\overleftrightarrow{\partial}_\lambda\pi_A+\sigma_B\overleftrightarrow{\partial}_\lambda\pi_B
\end{aligned} \tag{25}$$

Thus these currents are, respectively, dominated by π_A, σ_A, and π_- with each one of these Goldstone bosons possessing a tree approximation coupling to its associated current of strength p, to reconfirm that $\rho=1$ in tree approximation. Translating to the new minimum also induces new trilinear

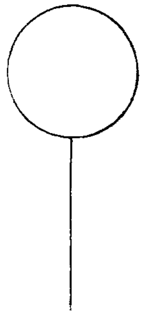


Fig. 1. One-loop tadpole contribution to the vacuum expectation value of the Higgs field.

couplings, viz.,

$$V_{\text{tri}}(\chi)=4(b+c)p\sigma_-[\sigma_+^2+\sigma_-^2+\pi_+^2+\pi_-^2+\sigma_A^2+\sigma_B^2+\pi_A^2+\pi_B^2]$$
$$-4(c-r)p\sigma_+[\sigma_+\sigma_- -\pi_+\pi_- -\sigma_A\sigma_B+\pi_A\pi_B]$$
$$-4(c+r)p\pi_+[\sigma_+\pi_- +\sigma_-\pi_+ -\sigma_A\pi_B-\sigma_B\pi_A] \tag{26}$$

so we are now able to compute the radiative corrections.

In one loop σ_- acquires the tadpole contribution of Figure 1 through the induced trilinear vertices. We restabilize the vacuum by adding a chiral-invariant counterterm to the Lagrangian. After translating it takes the form

$$\mathscr{L}_c=\tfrac{1}{2}A[\sigma_-^2+2p\sigma_-+p^2+\sigma_+^2+\pi_+^2+\pi_-^2+\sigma_A^2+\sigma_B^2+\pi_A^2+\pi_B^2] \tag{27}$$

and induces the graph of Figure 2. The choice

$$A=\frac{(b+c)}{8\pi^2}\left[10\Lambda^2-3M^2(\sigma_-)\ln\frac{\Lambda^2}{M^2(\sigma_-)}\right.$$
$$\left.-M^2(\sigma_+)\ln\frac{\Lambda^2}{M^2(\sigma_+)}-M^2(\pi_+)\ln\frac{\Lambda^2}{M^2(\pi_+)}\right]$$
$$-\frac{(c-r)}{8\pi^2}\left[\Lambda^2-M^2(\sigma_+)\ln\frac{\Lambda^2}{M^2(\sigma_+)}\right]-\frac{(c+r)}{8\pi^2}\left[\Lambda^2-M^2(\pi_+)\ln\frac{\Lambda^2}{M^2(\pi_+)}\right] \tag{28}$$

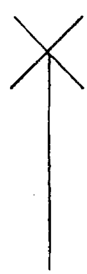


Fig. 2. Linear counterterm which restabilizes the vacuum.

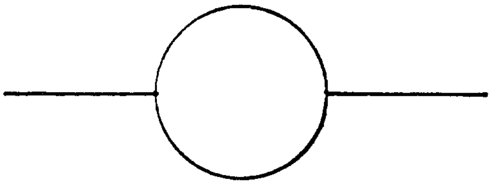


Fig. 3 Contribution of the trilinear couplings to the Goldstone boson propagator in one loop.

then enables Figure 1 to cancel against Figure 2 (both infinite and finite parts) so that $\langle\sigma_-\rangle$ remains equal to p. At the same time we find that the tadpole graphs for σ_+ and all the other fields all vanish identically. In one loop the Goldstone boson propagators acquire both the trilinear contributions of Figure 3 and the quartic contributions of Figure 4. Though these contributions are divergent we find that the counterterm of equation (27) induces just the right amount in Figure 5 to not only cancel these divergences but to also bring the masses of all of the five π_A, σ_A, π_-, π_B, and σ_B back to zero. Thus one counterterm is sufficient to maintain the Goldstone theorem in one loop.

The Goldstone boson propagators also undergo wave-function renormalization. Since only Figure 3 involves nonzero momentum entering the loop and since its graphs are only logarithmically divergent, the wave-function renormalization is completely finite. We shall refer to the renormalization constants of π_A, σ_A, and π_- as Z_1, Z_2, and Z_3, respectively. Evaluating through one-loop order then gives

$$Z_1^{-1}=Z_2^{-1}=1+\frac{(b+c)^2p^2}{\pi^2M^2(\sigma_-)}+\frac{(c-r)^2p^2}{4\pi^2M^2(\sigma_+)}+\frac{(c+r)^2p^2}{4\pi^2M^2(\pi_+)} \tag{29}$$

and

$$Z_3^{-1}=1+\frac{(b+c)^2p^2}{\pi^2M^2(\sigma_-)}+\frac{r^2p^2}{\pi^2[M^2(\sigma_+)-M^2(\pi_+)]^3} \times\left[M^4(\sigma_+)-M^4(\pi_+)-2M^2(\sigma_+)M^2(\pi_+)\ln\frac{M^2(\sigma_+)}{M^2(\pi_+)}\right] \tag{30}$$

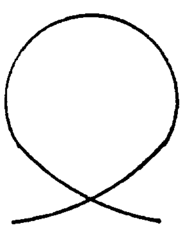


Fig. 4. Contribution of the quartic couplings to the Goldstone boson propagator in one loop.

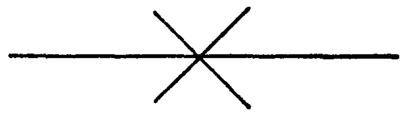


Fig. 5. Goldstone boson mass renormalization counterterm.

Using equation (24) for the explicit mass values gives

$$Z_1^{-1} = Z_2^{-1} = 1 + \frac{b}{8\pi^2}$$

$$Z_3^{-1} = 1 + \frac{b}{8\pi^2} + \frac{(c^2 - r^2)}{16\pi^2 r} \ln\left(\frac{c+r}{c-r}\right) \tag{31}$$

so that Z_1 and Z_3 are indeed different, as is to be expected in the absence of any residual symmetry.

In order to determine the gauge boson mass shifts we calculate the vacuum polarizations. We note that if the source of a gauge boson is J^i_λ, after translating the source will become $J^i_\lambda + p\partial_\lambda L_i$. Consequently in one loop the vacuum polarization receives contributions from the graphs of Figures 6–9. The contributions of Figure 6 are due to the renormalization of the Goldstone boson propagator, while those of Figures 7 and 8 involve vertex corrections to the coupling of the Goldstone bosons to the currents. The Goldstone boson pole terms of Figures 6–8 all contribute to the $q_\mu q_\nu / q^2$ term of the vacuum polarization while Figure 9 contributes to the $g_{\mu\nu}$ term to restore current conservation. The pole term contributions of Figures 6–8 are readily calculated. For π_A the tree approximation pole contribution to the vacuum polarization of $(V_1 - A_1)$ is

$$\pi^1_{\mu\nu} = -p^2 \frac{q_\mu q_\nu}{q^2} \tag{32}$$

while the one-loop contribution due to the renormalization of the π_A propagator in Figure 6 gives an additional contribution

$$\pi^1_{\mu\nu} = -p^2 \frac{q_\mu q_\nu}{q^2}(1 - Z_1^{-1}) \tag{33}$$

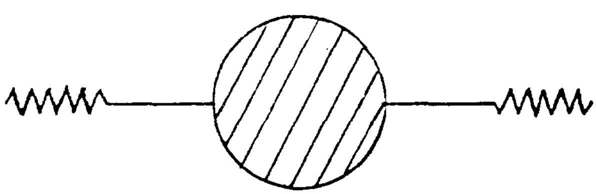
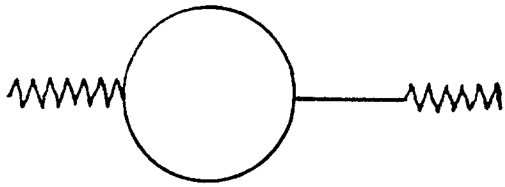
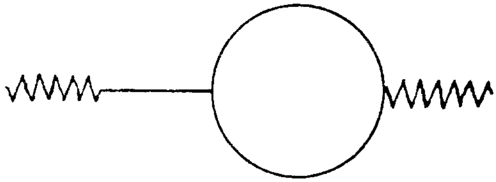


Fig. 6. Goldstone boson propagator renormalization contribution to the vacuum polarization. The shaded blob contains the graphs of Figures 3, 4, and 5.





Figs. 7 and 8. Contribution to vacuum polarization due to the vertex correction to the coupling of a Goldstone boson to its associated current.

The vertex corrections of Figures 7 and 8 are each found to yield

$$\pi^1_{\mu\nu}=p^2\frac{q_\mu q_\nu}{q^2}(1-Z_1^{-1}) \tag{34}$$

There is thus a Ward identity equivalence between the Goldstone boson wave function renormalization and its current vertex correction. [This provides a nice check on our explicit calculations since the Feynman diagrams in Figures 3 and 7 that we calculated to obtain equations (29) and (34), respectively, are completely different.] Then adding together the tree graphs and the one-loop contributions of Figures 6–8 yields a pole term

$$\pi^1_{\mu\nu}=-p^2\frac{q_\mu q_\nu}{q^2}Z_1^{-1} \tag{35}$$

so that

$$M^2(W_1^L)=g^2p^2Z_1^{-1} \tag{36}$$

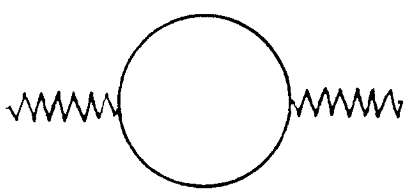


Fig. 9. Non-Goldstone boson contributions to the vacuum polarization.

A similar analysis for the vacuum polarization of $(V_3 - A_3)$ yields

$$\pi^3_{\mu\nu} = -p^2 \frac{q_\mu q_\nu}{q^2} Z_3^{-1} \tag{37}$$

so that

$$\cos^2 \theta_W M^2(Z_L) = g^2 p^2 Z_3^{-1} \tag{38}$$

With Z_1 and Z_3 given in equations (31) we thus find that ρ deviates from 1 by an amount dependent on the unknown parameters of the potential. We note that the above radiative corrections actually numerically only yield a 1% or so deviation for ρ from 1 in the event that the dimensionless parameters b, c, and r are themselves of order 1. Even though this may not be an unrealistic expectation, the basic difficulty with it is that we have no control on the values of the parameters. Consequently the present model has no apparent rationale for having ρ near 1, and we shall instead turn to another model in which the radiative corrections are under control.

4. LOCAL CHIRAL WEAK INTERACTIONS IN THE TREE APPROXIMATION

We begin our analysis with some general remarks on symmetry breaking in local chiral $SU(2)_L \times SU(2)_R \times (B-L)$ gauge theories. The gauge boson sector is given by

$$\begin{aligned}\mathcal{L} = {} & gW_1^L(V_1 - A_1) + gW_2^L(V_2 - A_2) + gW_3^L(V_3 - A_3) \\ & + gW_1^R(V_1 + A_1) + gW_2^R(V_2 + A_2) + gW_3^R(V_3 + A_3) \\ & + g' W_0[J_{\text{em}} - (V_3 - A_3) - (V_3 + A_3)]\end{aligned} \tag{39}$$

Here W_i^R $(i = 1, 2, 3)$ form an $SU(2)_R$ triplet of vector bosons to accompany the W_i^L of Section 2, and W_0 couples to the $(B-L)$ current of equation (3). It is convenient (Mannheim, 1979, 1980) to define a new basis for the neutral sector

$$\begin{aligned} A &= -\sin \theta_W \left(W_3^L + W_3^R + \frac{g}{g'} W_0 \right) \\ Z_L &= \cos \theta_W W_3^L - \sin \theta_W \tan \theta_W \left(W_3^R + \frac{g}{g'} W_0 \right) \\ Z_R &= \tan \theta_W \left(\frac{g}{g'} W_3^R - W_0 \right) \end{aligned} \tag{40}$$

where we have introduced the Weinberg angle this time via

$$\tan \theta_W = \frac{-g'}{(g^2+g'^2)^{1/2}} \tag{41}$$

In terms of these basis states we can rewrite the couplings of the gauge bosons to the currents in the form

$$\begin{aligned}
\mathscr{L} &= gW_1^L(V_1-A_1)+gW_1^R(V_1+A_1) \\
&\quad +gW_2^L(V_2-A_2)+gW_2^R(V_2+A_2) \\
&\quad -[g\sin\theta_W(A+\tan\theta_W Z_L)+g'\tan\theta_W Z_R]J_{\text{em}} \\
&\quad +[g\sec\theta_W Z_L+g'\tan\theta_W Z_R](V_3-A_3) \\
&\quad +g'\cot\theta_W Z_R(V_3+A_3)
\end{aligned} \tag{42}$$

After the symmetry is broken down to electromagnetism the theory will produce a total of six Goldstone bosons. They will be the three L_i introduced previously and three new R_i which couple to the right-handed currents as

$$\langle 0|(V_i+A_i)_\lambda|R_i\rangle = iq_\lambda F_i^R \tag{43}$$

Now in general the L_i Goldstone bosons can also couple to V_i+A_i while the R_i bosons can also couple to V_i-A_i, with respective strengths

$$\begin{aligned}
\langle 0|(V_i+A_i)_\lambda|L_i\rangle &= iq_\lambda P_i \\
\langle 0|(V_i-A_i)_\lambda|R_i\rangle &= iq_\lambda Q_i
\end{aligned} \tag{44}$$

This in general the Higgs mechanism yields a gauge boson mass matrix

$$\begin{aligned}
M &= \tfrac{1}{2}g^2(W_1^L)^2(F_1^L)^2+\tfrac{1}{2}g^2(W_1^R)^2(F_1^R)^2+g^2W_1^LW_1^R(F_1^LP_1+F_1^RQ_1) \\
&\quad +\tfrac{1}{2}g^2(W_2^L)^2(F_2^L)^2+\tfrac{1}{2}g^2(W_2^R)^2(F_2^R)^2+g^2W_2^LW_2^R(F_2^LP_2+F_2^RQ_2) \\
&\quad +\tfrac{1}{2}(g\sec\theta_W Z_L+g'\tan\theta_W Z_R)^2(F_3^L)^2+\tfrac{1}{2}g'^2\cot^2\theta_W Z_R^2(F_3^R)^2 \\
&\quad +(g\sec\theta_W Z_L+g'\tan\theta_W Z_R)g'\cot\theta_W Z_R(F_3^LP_3+F_3^RQ_3)
\end{aligned} \tag{45}$$

As can be seen from equations (39) and (45) breaking only according to the Dirac mass term χ does not by itself lead to a mass matrix in which W_1^L, W_2^L, and Z_L are light. (Rather, the pseudoscalar Goldstone bosons couple to the axial vector currents to diagonalize the gauge boson mass matrix in the $W_i^L-W_i^R$ basis.) Thus in local chiral theories of the weak interactions the analysis of Susskind (1979) and Weinberg (1976, 1979) does not by itself lead to anything like the Weinberg–Salam phenomenology. To proceed further we instead note from equation (45) that in the event that the F_i^R are very large the gauge bosons W_1^R, W_2^R, and Z_R will acquire large masses to leave over an approximate low-lying $SU(2)_L \times U(1)_{WS}$ sector with mass matrix

$$M = \tfrac{1}{2}g^2(W_1^L)^2(F_1^L)^2 + \tfrac{1}{2}g^2(W_2^L)^2(F_2^L)^2 + \tfrac{1}{2}g^2 \sec^2\theta_W Z_L^2(F_3^L)^2 \qquad (46)$$

Then as in Section 2

$$\rho = \frac{F_1^L}{F_3^L} \qquad (47)$$

only this time up to a small amount of mixing between the left- and right-handed gauge bosons. Thus equation (47) can also emerge in local chiral theories, only now as an approximate rather than an exact relation.

An explicit breaking pattern which realizes equation (46) has recently been presented in the literature (Mannheim, 1979, 1980), and it is a breaking which is also based on fermion bilinears, namely, Majorana mass breaking. Fermion Majorana masses transform as $\Delta_R = (\mathbf{1}, \mathbf{3}, -\mathbf{2})$ and $\Delta_L = (\mathbf{3}, \mathbf{1}, -\mathbf{2})$ under $SU(2)_L \times SU(2)_R \times (B-L)$. Thus difermion states carry a definite helicity (in contrast to Dirac masses which contain both left- and right-handed helicities), and also carry two units of fermion number (the particle-antiparticle Dirac mass has zero fermion number). Of the available difermion states there is only one electrically neutral one, the dineutrino, whose $B-L$ quantum number is -2. If we gave a vacuum expectation value to a fundamental Higgs field which transforms like a dineutrino Majorana mass Δ_R under the chiral group we then break parity, and also break $SU(2)_R$ and lepton number, but do not break $SU(2)_L$ or electric charge. Hence, according to equations (2) and (3), we precisely break the chiral group down to the Weinberg–Salam group. Thus right-handed Majorana mass breaking reduces the local chiral theory to a low-lying Weinberg–Salam symmetry in a compact and very elegant manner. Thus a large value for $\langle\Delta_R\rangle$ leads to large values for the F_i^R of equation (43). Finally, with the right-handed sector now heavy, the subsequent effect of χ is to then give masses to the Weinberg–Salam gauge bosons while only mixing them a little with the heavier right-handed gauge bosons. Thus if $\langle\Delta_R\rangle$ is very much greater than $\langle\chi\rangle$ we will recover equations (46) and (47) to order $\langle\chi\rangle^2/\langle\Delta_R\rangle^2$.

We note that while the above general analysis has the desired structure the actual value of ρ is not yet constrained, and ρ could in general be very different from 1. To proceed further requires an explicit model. Because the Δ_R term reduces the theory to an approximate Weinberg–Salam model we see immediately that a fundamental Higgs field χ will act in the tree approximation just as it did in the model studied in Section 2. Hence we will again find that F_1^L and F_3^L are equal in tree approximation for any values of $\langle\sigma_0\rangle$ and $\langle\sigma_3\rangle$. A convenient way to show this is to construct the gauge boson mass matrix via minimal coupling for fundamental Higgs fields

Δ_R, Δ_L, and χ. Being products of fundamentals these fields have straightforward transformation properties which are best illustrated in a tensor notation. We shall use Latin indices $(a, b \ldots = 1, 2)$ to describe $SU(2)_L$ tensors and Greek indices $(\alpha, \beta, \ldots = 1, 2)$ to describe $SU(2)_R$ tensors. In a tensor notation the gauge-covariant derivatives are

$$\begin{aligned}
&\partial_\lambda \Delta^R_{\alpha\beta} - ig(W^R_{\alpha\gamma})_\lambda \Delta^R_{\gamma\beta} - ig(W^R_{\beta\gamma})_\lambda \Delta^R_{\alpha\gamma} + 2ig'(W_0)_\lambda \Delta^R_{\alpha\beta}, \\
&\partial_\lambda \Delta^L_{ab} - ig(W^L_{ac})_\lambda \Delta^L_{cb} - ig(W^L_{bc})_\lambda \Delta^L_{ac} + 2ig'(W_0)_\lambda \Delta^L_{ab}, \\
&\partial_\lambda \chi_{a\alpha} - ig(W^L_{ab})_\lambda \chi_{b\alpha} + ig(W^R_{\beta\alpha})_\lambda \chi_{a\beta}
\end{aligned} \tag{48}$$

Allowing the electrically neutral Δ^R_{11}, Δ^L_{11}, χ_{11}, and χ_{22} to acquire vacuum expectation values gives [after using equation (12)] a tree approximation gauge boson mass matrix

$$\begin{aligned}
M = {} & g^2 \langle \Delta^R_{11} \rangle^2 \left[(W^R_1)^2 + (W^R_2)^2 + 2\left(W^R_3 - \frac{g'}{g} W_0 \right)^2 \right] \\
& + g^2 \langle \Delta^L_{11} \rangle^2 \left[(W^L_1)^2 + (W^L_2)^2 + 2\left(W^L_3 - \frac{g'}{g} W_0 \right)^2 \right] \\
& + \tfrac{1}{2} g^2 \langle \sigma_0 \rangle^2 [(W^L_1 - W^R_1)^2 + (W^L_2 - W^R_2)^2 + (W^L_3 - W^R_3)^2] \\
& + \tfrac{1}{2} g^2 \langle \sigma_3 \rangle^2 [(W^L_1 + W^R_1)^2 + (W^L_2 + W^R_2)^2 + (W^L_3 - W^R_3)^2]
\end{aligned} \tag{49}$$

Then in the limit

$$\langle \Delta^R_{11} \rangle \gg (\langle \sigma_0 \rangle, \langle \sigma_3 \rangle) \gg \langle \Delta^L_{11} \rangle \tag{50}$$

we obtain all the usual Weinberg–Salam phenomenology (Mannheim, 1979, 1980) with

$$\rho = 1 + O\left(\frac{\langle \sigma_0 \rangle^2}{\langle \Delta_R \rangle^2}, \frac{\langle \sigma_3 \rangle^2}{\langle \Delta_R \rangle^2}, \frac{\langle \Delta_L \rangle^2}{\langle \sigma_0 \rangle^2}, \frac{\langle \Delta_L \rangle^2}{\langle \sigma_3 \rangle^2} \right) \tag{51}$$

Reexpressing ρ as

$$\rho = 1 + O\left(\frac{M^2(W^L_1)}{M^2(W^R_1)} \right) = 1 + O\left(\frac{M^2(Z_L)}{M^2(Z_R)} \right) \tag{52}$$

we see that in the tree approximation to a chiral theory ρ only deviates from 1 by a small controllable factor dependent on the relative strengths of the charged or the neutral left- and right-handed currents of the theory. This small factor is controllable in the sense that it depends on the dimensionful parameters of the symmetry-breaking potential rather than on the dimensionless ones, so it is directly fixed by the symmetry-breaking scales of the theory. Thus the breaking pattern of equation (50) which we already need to control the charged sector of the theory will then automatically control the neutral sector as well according to equation (52).

Noting, finally, that we can obtain equation (52) above for any values of $\langle\sigma_0\rangle$ and $\langle\sigma_3\rangle$, we see that the fermion masses are unconstrained and free to be unequal. Moreover, with the fermion masses being unequal, the theory will produce more Goldstone bosons than can be absorbed by the $SU(2)_L \times U(1)_{WS}$ gauge bosons alone, a difficulty we alluded to earlier. We now see that in the local chiral theory we have just the right number of gauge bosons to remove these additional Goldstone bosons, since the symmetry of the gauge sector is the same as that of the potential. In this case there can be no residual global symmetry following the breaking, and hence we obtain equation (52) without a residual symmetry at all. Thus ρ will be close to 1 in the tree approximation to a chiral theory provided only that the breaking pattern satisfies equation (50). In order to take advantage of this nice result we now study the stability of our analysis against radiative corrections.

5. RADIATIVE CORRECTIONS TO LOCAL CHIRAL WEAK INTERACTION MODELS

In this section we investigate the stability of the tree approximation structure of the local chiral theory against radiative corrections. Rather than perturb around the most general tree approximation minimum we shall, just as in our discussion in Section 3, be able to obtain the result of interest by picking a simpler minimum. We consider a fundamental Higgs model with fields χ, Δ_R, and Δ_L. The most general remormalizable $SU(2)_L \times SU(2)_R \times (B-L)$ invariant potential for these fields is

$$\begin{aligned} V(\chi, \Delta) = V(\chi) &- d(\mathrm{Tr}\,\Delta_L^+\Delta_L + \mathrm{Tr}\,\Delta_R^+\Delta_R) \\ &+ e[(\mathrm{Tr}\,\Delta_L^+\Delta_L)^2 + (\mathrm{Tr}\,\Delta_R^+\Delta_R)^2] + f[\mathrm{Tr}(\Delta_L^+\Delta_L)^2 + \mathrm{Tr}(\Delta_R^+\Delta_R)^2] \\ &+ h(\mathrm{Tr}\,\Delta_L^+\Delta_L)(\mathrm{Tr}\,\Delta_R^+\Delta_R) + k[\mathrm{Tr}\,\chi^+\chi\Delta_L^+\Delta_L + \mathrm{Tr}\,\chi\chi^+\Delta_R^+\Delta_R] \\ &+ l(Tr\,\chi^+\chi)[\mathrm{Tr}\,\Delta_L^+\Delta_L + \mathrm{Tr}\,\Delta_R^+\Delta_R] \end{aligned} \quad (53)$$

where $V(\chi)$ is given in equation (17). For χ we shall again use the basis of equation (22), while for Δ_L we define

$$\begin{aligned} \Delta_{11}^L &= \gamma_L + i\delta_L \\ \Delta_{12}^L &= \Delta_{21}^L = \frac{1}{\sqrt{2}}(\tau_L + i\varepsilon_L) \\ \Delta_{22}^L &= \alpha_L + i\beta_L \end{aligned} \quad (54)$$

and analogously for Δ_R. The left-handed currents associated with χ are already given in equation (25). The additional currents associated with Δ_L are

$$
\begin{aligned}
(V_1-A_1)_\lambda &= \sqrt{2}[-\gamma_L\overleftrightarrow{\partial}_\lambda\varepsilon_L+\delta_L\overleftrightarrow{\partial}_\lambda\tau_L-\alpha_L\overleftrightarrow{\partial}_\lambda\varepsilon_L+\beta_L\overleftrightarrow{\partial}_\lambda\tau_L]\\
(V_2-A_2)_\lambda &= \sqrt{2}[\gamma_L\overleftrightarrow{\partial}_\lambda\tau_L+\delta_L\overleftrightarrow{\partial}_\lambda\varepsilon_L-\alpha_L\overleftrightarrow{\partial}_\lambda\tau_L-\beta_L\overleftrightarrow{\partial}_\lambda\varepsilon_L]\\
(V_3-A_3)_\lambda &= 2[-\gamma_L\overleftrightarrow{\partial}_\lambda\delta_L+\alpha_L\overleftrightarrow{\partial}_\lambda\beta_L]
\end{aligned}
\tag{55}
$$

The right-handed currents associated with χ are

$$
\begin{aligned}
(V_1+A_1)_\lambda &= \sigma_+\overleftrightarrow{\partial}_\lambda\pi_A-\pi_+\overleftrightarrow{\partial}_\lambda\sigma_A+\sigma_-\overleftrightarrow{\partial}_\lambda\pi_B-\pi_-\overleftrightarrow{\partial}_\lambda\sigma_B\\
(V_2+A_2)_\lambda &= \sigma_+\overleftrightarrow{\partial}_\lambda\sigma_A+\pi_+\overleftrightarrow{\partial}_\lambda\pi_A-\sigma_-\overleftrightarrow{\partial}_\lambda\sigma_B-\pi_-\overleftrightarrow{\partial}_\lambda\pi_B\\
(V_3+A_3)_\lambda &= \sigma_+\overleftrightarrow{\partial}_\lambda\pi_+-\sigma_-\overleftrightarrow{\partial}_\lambda\pi_--\sigma_A\overleftrightarrow{\partial}_\lambda\pi_A+\sigma_B\overleftrightarrow{\partial}_\lambda\pi_B
\end{aligned}
\tag{56}
$$

and those associated with Δ_R are

$$
\begin{aligned}
(V_1+A_1)_\lambda &= \sqrt{2}[-\gamma_R\overleftrightarrow{\partial}_\lambda\varepsilon_R+\delta_R\overleftrightarrow{\partial}_\lambda\tau_R-\alpha_R\overleftrightarrow{\partial}_\lambda\varepsilon_R+\beta_R\overleftrightarrow{\partial}_\lambda\tau_R]\\
(V_2+A_2)_\lambda &= \sqrt{2}[\gamma_R\overleftrightarrow{\partial}_\lambda\tau_R+\delta_R\overleftrightarrow{\partial}_\lambda\varepsilon_R-\alpha_R\overleftrightarrow{\partial}_\lambda\tau_R-\beta_R\overleftrightarrow{\partial}_\lambda\varepsilon_R]\\
(V_3+A_3)_\lambda &= 2[-\gamma_R\overleftrightarrow{\partial}_\lambda\delta_R+\alpha_R\overleftrightarrow{\partial}_\lambda\beta_R]
\end{aligned}
\tag{57}
$$

We seek a minimum in which we translate σ_- by p and γ_R by t (i.e., we simplify by taking $\langle\sigma_0\rangle=-\langle\sigma_3\rangle$ and $\langle\gamma_L\rangle=0$). The vanishing of the terms linear in the fields in the translated theory requires

$$
\begin{aligned}
[4(b+c)(e+f)-l^2]p^2 &= 2(e+f)a-ld\\
[4(b+c)(e+f)-l^2]t^2 &= 2(b+c)d-la
\end{aligned}
\tag{58}
$$

and has real solutions for p^2 and t^2 if $4(b+c)(e+f)>l^2$. Further, t^2 will be much larger than p^2 if $d\gg a$ and $(b+c)d\gg(e+f)a$. In the translated theory the complete quadratic term in the potential takes the exact form

$$
\begin{aligned}
V_{\text{quad}}(\chi,\Delta) &= \tfrac{1}{2}M_R^2R^2+\tfrac{1}{2}M_S^2S^2+(kt^2-2cp^2+2rp^2)\sigma_+^2\\
&\quad+(kt^2-2cp^2-2rp^2)\pi_+^2+\tfrac{1}{2}k(p^2+2t^2)(J^2+L^2)\\
&\quad+(kp^2-2ft^2)(\alpha_R^2+\beta_R^2)+(h-2e-2f)t^2(\gamma_L^2+\delta_L^2)\\
&\quad+[(h-2e-2f)t^2+\tfrac{1}{2}kp^2](\varepsilon_L^2+\tau_L^2)\\
&\quad+[(h-2e-2f)t^2+kp^2](\alpha_L^2+\beta_L^2)
\end{aligned}
\tag{59}
$$

Here the mass eigenstates are defined as

$$\begin{aligned} J &= (p\tau_R + \sqrt{2}\, t\sigma_B)/(p^2+2t^2)^{1/2} \\ L &= (p\varepsilon_R + \sqrt{2}\, t\pi_B)/(p^2+2t^2)^{1/2} \\ R &= \cos\theta\sigma_- + \sin\theta\gamma_R \\ S &= -\sin\theta\sigma_- + \cos\theta\gamma_R \end{aligned} \tag{60}$$

where

$$\tan 2\theta = \frac{lpt}{[(b+c)p^2 - (e+f)t^2]} \tag{61}$$

and the R, S mass eigenvalues are

$$\begin{aligned} M_R^2 &= 4(b+c)p^2 + 4(e+f)t^2 + 4\{[(b+c)p^2 - (e+f)t^2]^2 + l^2p^2t^2\}^{1/2} \\ M_S^2 &= 4(b+c)p^2 + 4(e+f)t^2 - 4\{[(b+c)p^2 - (e+f)t^2]^2 + l^2p^2t^2\}^{1/2} \end{aligned} \tag{62}$$

All of the 14 fields in equation (59) will have positive squared masses if, essentially, b, k, and h are large, so again our minimum is natural. There are 6 Goldstone bosons produced in the minimum, namely, σ_A, π_A, π_-, δ_R, M and N, where

$$\begin{aligned} M &= (-\sqrt{2}\, t\tau_R + p\sigma_B)/(p^2+2t^2)^{1/2} \\ N &= (-\sqrt{2}\, t\varepsilon_R + p\pi_B)/(p^2+2t^2)^{1/2} \end{aligned} \tag{63}$$

This is just the set needed to break the theory down to electromagnetism.

From equations (25), (56), and (57) we identify the translated parts of the currents in the tree approximation minimum as

$$\begin{aligned} (V_1 - A_1)_\lambda &\sim -p\partial_\lambda \pi_A, & (V_1 + A_1)_\lambda &\sim (p^2+2t^2)^{1/2}\partial_\lambda N \\ (V_2 - A_2)_\lambda &\sim -p\partial_\lambda \sigma_A, & (V_2 + A_2)_\lambda &\sim -(p^2+2t^2)^{1/2}\partial_\lambda M \\ (V_3 - A_3)_\lambda &\sim p\partial_\lambda \pi_-, & (V_3 + A_3)_\lambda &\sim -p\partial_\lambda \pi_- - 2t\partial_\lambda \delta_R \end{aligned} \tag{64}$$

Hence from equation (45) we obtain a gauge boson mass matrix

$$\begin{aligned} M = {} & \tfrac{1}{2}g^2p^2[(W_1^L)^2 + (W_2^L)^2] + \tfrac{1}{2}g^2(p^2+2t^2)[(W_1^R)^2 + (W_2^R)^2] \\ & + \tfrac{1}{2}(g\sec\theta_W Z_L + g'\tan\theta_W Z_R)^2p^2 + \tfrac{1}{2}g'^2\cot^2\theta_W Z_R^2(p^2+4t^2) \\ & - (g\sec\theta_W Z_L + g'\tan\theta_W Z_R)g'\cot\theta_W Z_R p^2 \end{aligned} \tag{65}$$

Thus when $t^2 \gg p^2$ the Weinberg-Salam phenomenology indeed follows with $(\rho - 1)$ being of order p^2/t^2 in the tree approximation as required.

Translating to the tree approximation minimum also induces a host of trilinear couplings, which we write in the initial basis for convenience, viz.,

$$\begin{aligned}V_{\rm tri}(\chi,\Delta)=&\,V_{\rm tri}(\chi)+4(e+f)t\gamma_R[\gamma_R^2+\delta_R^2+\varepsilon_R^2+\tau_R^2+\alpha_R^2+\beta_R^2]\\&-4ft\gamma_R(\alpha_R^2+\beta_R^2)+4ft\beta_R\varepsilon_R\tau_R+2ft\alpha_R(\tau_R^2-\varepsilon_R^2)\\&+2h\gamma_Rt[\gamma_L^2+\delta_L^2+\varepsilon_L^2+\tau_L^2+\alpha_L^2+\beta_L^2]\\&+2lp\sigma_-[\gamma_L^2+\delta_L^2+\varepsilon_L^2+\tau_L^2+\alpha_L^2+\beta_L^2\\&+\gamma_R^2+\delta_R^2+\varepsilon_R^2+\tau_R^2+\alpha_R^2+\beta_R^2]\\&+2lt\gamma_R[\sigma_+^2+\sigma_-^2+\pi_+^2+\pi_-^2+\sigma_A^2+\sigma_B^2+\pi_A^2+\pi_B^2]\\&+kp\sigma_-[\varepsilon_L^2+\tau_L^2+2\alpha_L^2+2\beta_L^2]\\&+\sqrt{2}\,kp\sigma_A[(\gamma_L+\alpha_L)\tau_L+(\delta_L+\beta_L)\varepsilon_L]\\&-\sqrt{2}\,kp\pi_A[(\gamma_L-\alpha_L)\varepsilon_L+(\beta_L-\delta_L)\tau_L]\\&+kp\sigma_-[\varepsilon_R^2+\tau_R^2+2\alpha_R^2+2\beta_R^2]\\&+\sqrt{2}\,kp\sigma_B[(\gamma_R+\alpha_R)\tau_R+(\delta_R+\beta_R)\varepsilon_R]\\&+\sqrt{2}\,kp\pi_B[(\gamma_R-\alpha_R)\varepsilon_R+(\beta_R-\delta_R)\tau_R]\\&+2kt\gamma_R[\sigma_+^2+\pi_+^2+\sigma_B^2+\pi_B^2]\\&+\sqrt{2}\,kt\tau_R[\sigma_+\sigma_A+\sigma_-\sigma_B+\pi_+\pi_A+\pi_-\pi_B]\\&+\sqrt{2}\,kt\varepsilon_R[-\sigma_+\pi_A+\sigma_-\pi_B+\pi_+\sigma_A-\pi_-\sigma_B]\end{aligned}\tag{66}$$

Armed with this set of vertices we are now able to calculate the radiative corrections.

We calculate the wave-function renormalization constants from the appropriate one loop graphs of Figure 3. For the π_A propagator there are ten intermediate pairs which contribute, namely, (π_A, R), (π_A, S), (σ_+, N), (σ_+, L), (π_+, M), (π_+, J), $(\gamma_L, \varepsilon_L)$, (δ_L, τ_L), (β_L, τ_L) and $(\alpha_L, \varepsilon_L)$. Altogether they yield through one-loop order

$$\begin{aligned}Z_1^{-1}-1=\frac{1}{16\pi^2}\Bigg\{&\frac{4[2(b+c)p\cos\theta+lt\sin\theta]^2}{M_R^2}\\&+\frac{4[-2(b+c)p\sin\theta+lt\cos\theta]^2}{M_S^2}+\frac{(kt^2-2cp^2+2rp^2)^2}{(p^2+2t^2)M^2(\sigma_+)}\\&+\frac{p^2t^2(k+4c-4r)^2J(\sigma_+,L)}{2(p^2+2t^2)}+\frac{(kt^2-2cp^2-2rp^2)^2}{(p^2+2t^2)M^2(\pi_+)}\\&+\frac{p^2t^2(k+4c+4r)^2J(\pi_+,J)}{2(p^2+2t^2)}\\&+\tfrac{1}{2}k^2p^2[J(\gamma_L,\varepsilon_L)+J(\delta_L,\tau_L)+J(\beta_L,\tau_L)+J(\alpha_L,\varepsilon_L)]\Bigg\}\end{aligned}\tag{67}$$

where we use the symbol $J(x, y)$ to denote

$$J(x, y) = \frac{M_x^2 + M_y^2}{(M_x^2 - M_y^2)^2} - \frac{2M_x^2 M_y^2}{(M_x^2 - M_y^2)^3} \ln \frac{M_x^2}{M_y^2} \tag{68}$$

Inserting the masses given in equation (59) and taking the limit $t^2 \gg p^2$ yields

$$Z_1^{-1} = 1 + \frac{b+c}{8\pi^2} + \frac{k}{32\pi^2} + O\left(\frac{p^2}{t^2}\right) \tag{69}$$

The calculation for the π_- propagator is analogous. For π_- there are five relevant pairs, namely, (π_-, R), (π_-, S), (π_+, σ_+), (L, M), and (J, N). Through one-loop order they yield

$$\begin{aligned} Z_3^{-1} - 1 = \frac{1}{16\pi^2} \Bigg\{ & \frac{4[2(b+c)p \cos\theta + lt \sin\theta]^2}{M_R^2} \\ & + \frac{4[-2(b+c)p \sin\theta + lt \cos\theta]^2}{M_S^2} \\ & + 16r^2 p^2 J(\pi_+, \sigma_+) + \frac{k^2 t^2}{2M_L^2} + \frac{k^2 t^2}{2M_J^2} \Bigg\} \end{aligned} \tag{70}$$

Again using equation (59) we obtain in the $t^2 \gg p^2$ limit

$$Z_3^{-1} = 1 + \frac{b+c}{8\pi^2} + \frac{k}{32\pi^2} + O\left(\frac{p^2}{t^2}\right) \tag{71}$$

Hence we find that while Z_1 and Z_3 are in general unequal in one loop, in the $t^2 \gg p^2$ limit their leading terms are equal. Moreover, their nonleading terms only differ by the same controllable amount in one loop as ρ differs from 1 in the tree approximation.

Though the one-loop Goldstone boson mass matrix remains diagonal in the tree approximation basis (since the Goldstone bosons stay degenerate), we note that the wave-function renormalization constants have off-diagonal terms. Specifically, as can be inferred from the structure of $(V_3 + A_3)$ given in equation (64), there is a π_- to δ_R transition in one loop. Evaluating Figure 3 for intermediate pairs (J, N) and (L, M) yields

$$Z^{-1}(\pi_-, \delta_R) = -\frac{1}{32\pi^2}\left[\frac{k^2 pt}{M_J^2} + \frac{k^2 pt}{M_L^2}\right] \tag{72}$$

which is of order p/t in the large-t^2 limit.

To complete the calculation we evaluate the vertex correction graphs of Figures 7 and 8. We find that they give the same Z factors as those of equations (67) and (70) above. Since the Feynman diagrams are completely different from those of Figure 3 this provides an independent check on our calculations as we discussed in Section 3. Finally, adding all the pole terms of Figures 6–8 together gives us the complete one-loop modification to the gauge boson mass matrix of equation (65). The dominant contributions come from the modifications of the $\langle O|(V_i - A_i)(V_i - A_i)|0\rangle$ vacuum polarizations due to Z_1 and Z_3. The π_- to δ_R transition contributes to the $\langle O|(V_3 - A_3)(V_3 + A_3)|0\rangle$ vacuum polarization to give a term of order $p(p/t)t$, i.e. of order p^2. This modifies the coefficient of the $(g \sec\theta_W Z_L + g' \tan\theta_W Z_R)g' \cot\theta_W Z_R$ term of equation (65) by a factor of order p^2. The π_- to δ_R transition also modifies the coefficient of the $g'^2 \cot^2\theta_W Z_R^2$ term by a factor of order p^2. Thus the π_- to δ_R transition only modifies the Z_L, Z_R mixing which was already small in tree approximation. Hence for the low-lying Weinberg–Salam sector we find that equation (65) is modified to

$$M = \frac{g^2 p^2}{2}\left[1 + \frac{(b+c)}{8\pi^2} + \frac{k}{32\pi^2}\right][(W_1^L)^2 + (W_2^L)^2 + \sec^2\theta_W Z_L^2] \tag{73}$$

up to correction terms of order p^2/t^2, so that finally

$$\rho = 1 + O\left(\frac{p^2}{t^2}\right) \tag{74}$$

through one-loop order.

This is then our desired result with ρ being close to 1 and with the quarks being far from degenerate. It is of interest to identify why our model produces this at first surprising result. We recall that the symmetry of $V(\chi, \Delta)$ of equation (53) is only $SU(2)_L \times SU(2)_R \times (B-L)$ with the two real σ-model quartets α and β in χ being separately irreducible under $SU(2)_L \times SU(2)_R$. Inspection of the electrically neutral piece of the $k \operatorname{Tr} \chi\chi^\dagger \Delta_R^\dagger \Delta_R$ term of $V(\chi, \Delta)$, viz. [using the notation of equations (48) and (54)]

$$\begin{aligned}
&\operatorname{Tr} \chi\chi^\dagger \Delta_R^\dagger \Delta_R \\
&\quad = \sum_{\substack{a,\alpha \\ \beta,\gamma}} (\chi)_{a\alpha}(\chi^\dagger)_{\alpha\beta}(\Delta_R^\dagger)_{\beta\gamma}(\Delta_R)_{\gamma\alpha} \\
&\quad = \tfrac{1}{2}[2\gamma_R^2 + 2\delta_R^2 + \tau_R^2 + \varepsilon_R^2][\sigma_+^2 + \sigma_B^2 + \pi_B^2 + \pi_+^2] \\
&\qquad + \sqrt{2}[(\gamma_R + \alpha_R)\tau_R + (\delta_R + \beta_R)\varepsilon_R][\sigma_+\sigma_A + \sigma_-\sigma_B + \pi_+\pi_A + \pi_-\pi_B] \\
&\qquad + \sqrt{2}[(\gamma_R - \alpha_R)\varepsilon_R + (\beta_R - \delta_R)\tau_R][-\sigma_+\pi_A + \sigma_-\pi_B + \pi_+\sigma_A - \pi_-\sigma_B] \\
&\qquad + \tfrac{1}{2}[2\alpha_R^2 + 2\beta_R^2 + \tau_R^2 + \varepsilon_R^2][\sigma_-^2 + \sigma_A^2 + \pi_A^2 + \pi_-^2]
\end{aligned} \tag{75}$$

shows that at the same time as the γ_R piece of Δ_R breaks the chiral group down to $SU(2)_L \times U(1)_{WS}$ it also gives a large mass to a particular and unique linear combination of α and β, viz. [as exhibited in equation (59)],

$$\begin{aligned}\chi_B &= (1/\sqrt{2})(\sigma_0+\sigma_3, \sigma_1-\pi_2, \pi_1+\sigma_2, \pi_0+\pi_3)\\ &= (\sigma_+, \sigma_B, \pi_B, \pi_+)\end{aligned} \tag{76}$$

while leaving the orthogonal combination

$$\begin{aligned}\chi_A &= (1/\sqrt{2})(\sigma_0-\sigma_3, \sigma_1+\pi_2, \pi_1-\sigma_2, \pi_0-\pi_3)\\ &= (\sigma_-, \sigma_A, \pi_A, \pi_-)\end{aligned} \tag{77}$$

so far massless. [Essentially the neutrino like piece of Δ_R couples in the k term to $\sigma_0+\sigma_3$ ($\sim\bar{u}u$) and not to $\sigma_0-\sigma_3$ ($\sim\bar{d}d$).] Now as can be seen from equations (25) and (56) though χ_A and χ_B are separately irreducible under $SU(2)_L$, they are reducible under $SU(2)_R$ which mixes them. But, with χ_A being a real quartet of fields it must therefore be irreducible under some other $SU(2)$, $SU(2)_A$, say, with its self-couplings then being $SU(2)_A$ invariant, even though this $SU(2)_A$ is not an invariance of the input chiral Lagrangian. Since χ_A is the only light Higgs multiplet left in the theory following the Δ_R breaking, the effective low-energy symmetry of the potential is thus enlarged to $SU(2)_L\times U(1)_{WS}\times SU(2)_A$. Thus by giving a large mass to χ_B, the Δ_R breaking serves to establish an effective additional $SU(2)_A$ invariance in the light sector of the theory. Finally then, when we break the Weinberg–Salam group down to electromagnetism by giving the $\sigma_0-\sigma_3$ term in χ_A a vacuum expectation value we reduce the symmetry of the light sector of $SU(2)_L\times SU(2)_A$ to its "diagonal" $SU(2)_{L+A}$ subgroup. Under this residual symmetry the σ_A, π_A, and π_- Goldstone bosons transform as a triplet. With the three $SU(2)_L$ gauge bosons also transforming as a triplet under this $SU(2)_{L+A}$, we thus see that the all order pure χ_A radiative corrections leave ρ equal to 1. Further, with $\sigma_0-\sigma_3$ (which transforms as $\bar{d}d$) being an $SU(2)_{L+A}$ singlet this effective low-energy symmetry imposes no mass degeneracy on the fermions [$SU(2)_{L+R}$ was anyway already broken by Δ_R]. Thus the effective $SU(2)_{L+A}$ does just what is required in the light χ_A Higgs sector.

As well as being renormalized by χ_A, ρ is also renormalized by χ_B. As for the contribution of the heavy χ_B to ρ we note as follows. All the Goldstone bosons which couple to the Weinberg–Salam gauge bosons in the tree approximation are contained in χ_A [see, e.g., equation (64)]. Hence the renormalization of Z_1 and Z_3 due to χ_B is obtained from a χ_B loop contribution to the χ_A propagator (with typical diagrams such as those of Figure 6). Such diagrams exist in the translated theory because of induced tree level $\chi_A\chi_B^2$ trinlinear vertices. These trilinear couplings are of order $eM_A^2/M_W \sin\theta_W$, where M_A denotes the mass of a Higgs boson in the light χ_A. Hence the χ_B loop contribution to ρ gives an effect of order

$e^2 M_A^4 / M_W^2 M_B^2 \sin^2 \theta_W$, where M_B is a typical heavy χ_B mass. This effect is of order $\langle \chi_A \rangle^2 / \langle \Delta_R \rangle^2$ $(= p^2/t^2)$ so that equation (74) follows. The difference between this result and the pure $SU(2)_L \times U(1)_{WS}$ invariant two Higgs doublet model discussed in the Introduction is that now χ_B gets its mass not only from $\langle \chi_A \rangle$, but also (and predominantly) from the big $\langle \Delta_R \rangle$, while the Weinberg–Salam gauge bosons do not get their masses anymore from $\langle \chi_B \rangle$, but rather from $\langle \chi_A \rangle$. Indeed, were $M_B = M_A = M_H$ our result would reduce to the previously found large effect of order $e^2 M_H^2 / M_W^2 \sin^2 \theta_W$.

With regard to our analysis we note here the multiple role played by the right-handed neutrino Majorana mass Δ_R. First of course it produces the correct phenomenology in the tree approximation. Second, by giving χ_B a large mass it enables the heavy Higgs doublet to essentially decouple from the tree approximation Weinberg–Salam gauge boson mass matrix, and then from the radiative corrections to ρ. Third, by making χ_B heavy it also enables the χ_A self-couplings to enjoy their full $O(4)$ invariance and establish an effective custodial $SU(2)_{L+A}$ symmetry in the low-energy sector of the theory. Finally, since the $SU(2)_{L+A}$ symmetry is not an invariance of the initial Lagrangian, the $SU(2)_{L+A}$ symmetry is itself broken in the couplings of χ_A to χ_B, Δ_R and Δ_L. Since this is an effect of order $\langle \chi_A \rangle^2 / \langle \Delta_R \rangle^2$, equation (74) immediately follows.

With regard to phenomenological applications of our mechanism we find that not only is ρ close to 1 but its deviation from 1 is determined by an in principle controllable physical parameter since p^2/t^2 is the ratio of the strengths of the left-handed to right-handed charged currents, a ratio which is known to be small experimentally. Thus we correlate the ratio of the strengths of the charged to neutral Weinberg–Salam currents with the ratio of the strengths of the left-handed to right-handed charged chiral currents. Then simply because $SU(2)_L \times SU(2)_R \times (B-L)$ is broken down to its $SU(2)_L \times U(1)_{WS}$ subgroup at some large scale it follows that ρ is close to 1. Further, the deviation of ρ from 1 depends on the relative strengths of the dimensionalful scale parameters a and d of the potential $V(\chi, \Delta)$ and is not affected by magnitudes of the dimensionless quartic coefficients in the Higgs potential. Hence equation (74) will still hold even if the Higgs sector is strongly coupled, with the emergence of an effective low-energy $SU(2)_{L+A}$ symmetry only being sensitive to the values of the scale-breaking parameters and not to the magnitudes of the dimensionless quartic couplings. Finally then, because the value of ρ is only controlled by this physically small ratio of the dimensionful scale parameters, it is reasonable to expect that the multiloop radiative corrections will retain the effective $SU(2)_{L+A}$ invariance and continue to maintain equation (74) both in higher orders and also even in strong coupling.

In conclusion therefore we have shown that in a local $SU(2)_L \times SU(2)_R \times (B-L)$ left–right symmetric electroweak gauge theory right-handed neutrino Majorana mass breaking generates an effective low-energy custodial symmetry which is not an invariance of the input chiral Lagrangian. This effective custodial symmetry serves to maintain the relation $M_W = M_Z \cos \theta_W$ to any required degree of accuracy while leaving the fermion mass spectrum completely unconstrained.

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